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CENTRAL INTELLIGENCE AGENCY

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25 YEAR RE-REVIEW

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Symbol Designations and Formulation of the Equations of Motion to Calculate Missile Trajectories, Gama, and Ostashkov (1946)

11. To avoid repetitions the following basic symbol designations are herewith presented:

a (m/sec)	velocity of sound
A (kg)	lifting force
c (m/sec)	exhaust velocity of combustion gases
c_a (1)	lift coefficient
c_w (1)	resistance coefficient
d (m)	maximum diameter of the rocket
e (m)	distance from the starting point on the earth surface
e_1 (m)	distance of the pressure point (point of application of the listing force) from the nose of the rocket
e_2 (m)	distance of the rudder-pressure-center from the nose of the rocket
e_3 (m)	distance of the jet pressure point from the nose of the rocket
F (m^2)	maximum cross-section of the rocket
F_D (m^2)	end cross-section of the laval nozzle
g (m/sec^2)	acceleration of gravity
G (kg)	weight of the rocket
h (m)	altitude above the earth's surface
l (m)	length of the rocket
m ($kg\ sec^2/m$)	mass of the rocket
Ma (1)	Mach number
n (1)	load factor (lift:weight)
p (kg/m^2)	atmospheric pressure
P (kg/radian)	jet cross force per radian of the rudder deflection
q (kg/m^2)	dynamic pressure
r (m)	distance of a trajectory point from the launching point, sometimes also from the beam station.

R (m)	earth radius
s (m)	distance of the center of gravity from the nose of the rocket
S (kg)	thrust
t (sec)	time
T (°K)	temperature
v (m/sec)	velocity
W (kg)	air resistance
x (m)	horizontal distance in the tangential plane, through the launching point
y (m)	height over the tangential plane through starting point
z (m)	third coordinate vertically through x and y
α (degrees)	angle of incidence between path tangent and the longitudinal axis of the rocket
γ (degrees)	inclination angle of the flight path between x axis and path tangent
η (degrees)	rudder angle
ϑ (degrees)	angle between x axis and longitudinal axis of the rocket
μ (1)	mass ratio
ρ (kg sec^2/m^4)	air density
φ (degrees)	angle between vectors, center of the earth - launching point and center of the earth - path point of the rocket (Note: in general equations the angles are taken in circular measure).

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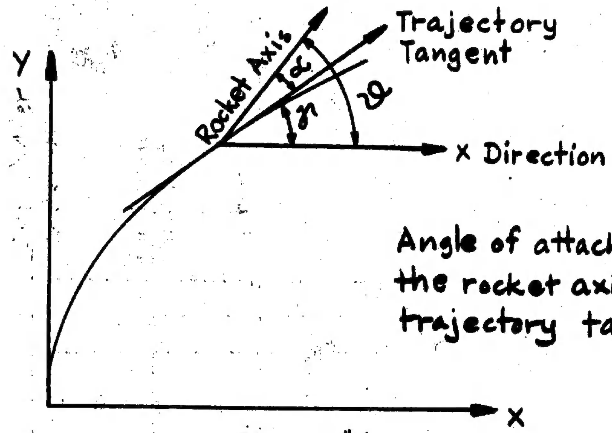
12. The plane powered trajectory:



Angle of attack α positive when the rocket axis is above the trajectory tangent $\vartheta = \alpha + \gamma$

Figure #1

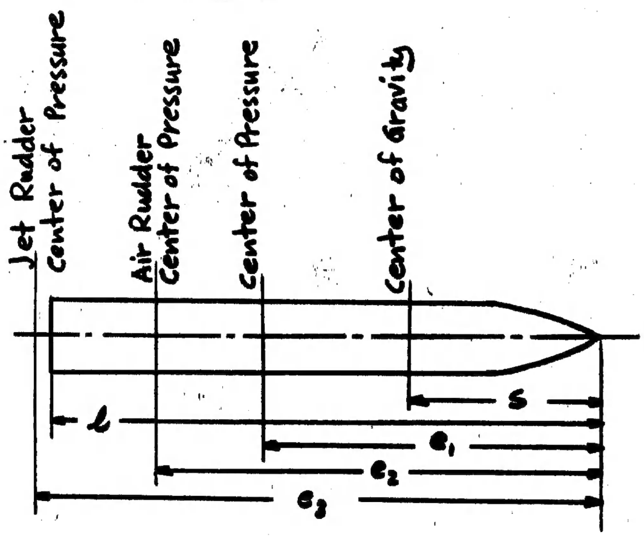


Figure #2

Meaning of the subscripts:

0 = Value at the start ($t = 0$) or on the ground ($h = 0$)

∞ = Value in a vacuum.

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13. For the ballistic calculations the motion equations were formulated as follows:

a. Force equation (in flight path direction):

$$m\dot{v} = S - W - mg \sin\gamma$$

b. Force equation (perpendicular to path direction):

$$mv\dot{\gamma} = S\alpha + qF\frac{\partial c_a}{\partial\alpha}\alpha + qF\frac{\partial c_a}{\partial\eta}\eta + P\eta - mg\cos\gamma$$

c. Moments equation:

$$(e_1 - s)qF\frac{\partial c_a}{\partial\alpha}\alpha + (e_2 - s)qF\frac{\partial c_a}{\partial\eta}\eta + (e_3 - s)P\eta = 0$$

14. In the first equation appear: thrust, whereby the component $S\cos\alpha$ is substituted by S, air resistance, and the weight component.

15. The thrust is composed of the thrust on the earth surface S_o, the thrust increase with altitude $P_oF_D(1-P/P_o)$, and, if there are jet rudders, the thrust-less through the jet rudders ΔS. For ΔS, a constant medium value was fixed independent of the rudder deflection.

16. Dr. WOLFF and Dr. ALBRING had established the mean value to use, probably by means of data from Peenemuende. ΔS was of the order of 12-15% of S_o for maximum rudder deflection of 25°.

17. For the thrust the equation is: $S = S_o + P_oF_D(1 - P/P_o) - \Delta S$,

or also emphasising the thrust in a vacuum ($S\infty$)

$$S = S_\infty - p_o F_D \frac{p}{p_o} - \Delta S.$$

18. The mass m is a linear function of the time: $m = m_o + \dot{m}t$ whereby $\dot{m}$ is negative. $\dot{m}$ was generally calculated as constant. In several A-4 calculations a variable weight rate of flow for the first four seconds was calculated until the full thrust was achieved.

19. Ground thrust and thrust in a vacuum were formulated as follows:

$$S = |\dot{m}/c_o \quad \text{and} \quad S_\infty = |\dot{m}/c_\infty$$

20. c_o and c_∞ were hereby exhaust speed of the gases on the ground and in a vacuum, respectively. Accordingly, altitude variable exhaust speed (c) was introduced by the relation $S = |\dot{m}/c$.

21. The air resistance has the form: $W = qFc_w$ with $q = \frac{\rho}{2}v^2 = \frac{\rho_o}{2}\frac{\rho}{\rho_o}v^2$.

22. The resistance factor c_w was given as a function of the Mach number $M_a = \frac{v}{c_o}\sqrt{T_o/T}$, the altitude h and for antiaircraft rockets of the angle of incidence α; for guided missiles the function of the angle of incidence was generally neglected for Eqt. (1) only.

23. For the functions $\sqrt{\frac{T_0}{T}}, \frac{p}{p_0}, \frac{\rho}{\rho_0}$ as a function of the altitude (h), a table existed, which used as basic data the values of the German artillery (normal atmosphere). For higher altitudes a temperature curve was taken, which Prof. SEELIGER (Greifswald) stated in a report which he wrote in 1945 for the Zentralwerke Bleicherode. The table covered an altitude up to 80 kilometers. Furthermore, the table contained the curve of g as a function of the altitude ($g_0 = 9.810$ met/sec^2).

24. (Since about 1950 another table of the atmosphere was used, in which data on the ground values is given by Schapiro, a temperature curve in low altitudes is taken from Russian literature (Schapiro), and for high altitudes a curve derived from foreign experimental results ([redacted]) were used. The literature was available in the branch library. The researcher for this table of the atmosphere was Dr. SCHLIER. [redacted] the new table covered up to 100 kilometers altitude. One of the reasons for making up a new table was that in the old table p/p_0 and ρ/ρ_0 were given to three valid places. This resulted in some cases by integrating in an irregular course of the differences. The new table was calculated for four valid places.) 25X1 25X1

25. In the second equation appears the thrust component, in which $\sin\alpha$ is replaced by α, the lift, the rudder forces and the weight component. In the lift coefficient $\partial c_a/\partial\alpha$ as a function of the Mach number and eventually of the altitude, $\partial c_a/\partial\eta$ as a function of the Mach number was given. These values were given by aerodynamics in such a form that the largest cross-section of the rocket was always taken as the datum plane. If the rocket had jet rudders, the resulting cross force was applied in form $P\eta$ with constant P.

26. In the moments equation, corresponding to the values in the second force equation, the factors $qF\frac{\partial c_a}{\partial\alpha}\alpha, qF\frac{\partial c_a}{\partial\eta}\eta$ and - if present $P\eta$ - were taken into consideration. The distances of the points of application of these forces from the center of gravity are $e_1 - s$, $e_2 - s$, and $e_3 - s$ respectively. The curve of (s) a function of time (t), or of the rocket mass was determined by the design section; the aerodynamics section supplied the curve of e_1 as a function of the Mach number; e_2 and e_3 are constant; e_3 was in general slightly larger than the length (l) of the rocket. The data for center of gravity and pressure center curve were as usual without dimensions in the form $\frac{s}{l}, \frac{e_1}{l}, \frac{e_2}{l}, \frac{e_3}{l}$.
For the ballistic calculations in every instance moment balance was assumed, so that the right side of the equation (3) was equal to zero. Therefore it was not necessary for ballistics to know the moment of inertia around the transverse axis of the rocket; in first place the complicating factors ϑ and ϑ in the mathematical treatment of the equation system were eliminated (ϑ = angle between rocket axis and horizontal plane).

27. For ballistic investigations no further factors were considered. The consideration of further moments and the process of stability examinations were the task of the guidance section (Dr. HOCH).

 [redacted] 25X1

28. HOCH used his "Bahnmodell" for the study of the second force equation and the moments equation, and also for the examination of the side stability. [redacted] 25X1

29. In the above formulation the equations 2 and 3 represent linear equations for the angles of incidence and the rudder angle η .

30. Equation (3) has the result:

(4) $\eta = -k\alpha$

with

(5) $k = \dfrac{(e_1-s)\,q\,F c_\alpha'}{(e_2-s)\,q\,F c_\eta' + (e_3-s)P}$ or $k = \dfrac{(e_1-s)\,c_\alpha'}{(e_2-s)\,c_\eta'}$

$$c_\alpha' = \frac{\partial c_a}{\partial \alpha}, \qquad c_\eta' = \frac{\partial c_a}{\partial \eta}$$

31. The second form of the equation (5) arises, if no jet rudders are included (P = 0).

32. From the equations (2 and 4) results

(6) $\alpha = \dfrac{m(v\dot{\gamma} + g\cos\gamma)}{N}$

with

(7) $N = S + qFc_\alpha' - (qFc_\eta' + P)k$, or $N = S + qFc_\alpha' \dfrac{e_2-e_1}{e_2-s}$

33. The second form of the equation (7) is again for P = 0.

34. To the equations (1) to (3) belong the equations for the trajectory coordinates:

(8) $\dot{x} = v\cos\gamma$

(9) $\dot{y} = v\sin\gamma$

35. At small distances from the launching point Y can be taken as altitudes (h) over the earth surface. At greater distances the difference between h and Y has to be considered:

(10) $h = y + \Delta h$

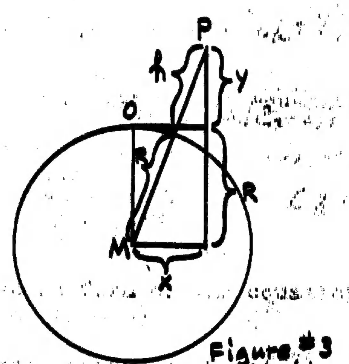

Figure #3

O = starting point (zero point of the x, y coordinate system)

M = center of the earth

P = trajectory point with coordinates x , y

R = radius of the earth = 6370 kilometers

36. From $(R+h)^2 = (R+Y)^2 + x^2$

and (10) disregarding $(\Delta h)^2$:

$$(R+Y)^2 + 2(R+Y)\Delta h = (R+Y)^2 + x^2$$

or (11) $\Delta h = \frac{x^2}{2(R+Y)} = \frac{x^2}{2R}\left(\frac{1}{1+Y/R}\right) \approx \frac{x^2}{2R}\left(1 - \frac{Y}{R}\right)$.

37. For powered flight path the approximation (11') $\Delta h \approx \frac{x^2}{2R}$ is generally sufficient.

38. For Δh as a function of X or of X and Y, tables were established.

39. In connection with the curvature of the earth, which determines the difference between h and y, it must be said, that gravity for greater distance from the zero point of the coordinates system has a noticeable x-component. When in the sketch Figure No. 3 the angle at P, which is equal to the angle OMP, is designated as φ, the gravity component is: $g_x = -g \sin \varphi = -g \frac{x}{R+h} \approx -g \frac{x}{R}$

$$g_y = -g \cos \varphi \approx -g$$

40. The consideration of g_x was done, if at all necessary, by a separate disturbance calculation. This was not done for the Wasserfall, but was considered for R-10, R-12, R-14 and V-2. Unknown factors in the equation (1) are: velocity v, altitude h, which appears in S, W and g, and the inclination angle of flight path γ. Through altitude equation (1) is coupled with equation (9), and - in case the difference between h and y has to be considered - also with equation (8), so that these differential equations must be integrated simultaneously.

41. The whole equation system has one more unknown factor than the number of equations. This gives the liberty to assume one condition.

42. The possibilities are the following:

   a. The inclination angle of flight path γ is prescribed.

   b. The angle of incidence α is prescribed.

   c. In antiaircraft rockets: the altitude angle Γ, under which the rocket appears from the starting point, is prescribed. In this case also the connection between Γ and the other values is needed.

43. If γ is prescribed, or fixed for other reasons, then equations (1), (8), (9), (11) represent four equations for the four unknown factors v, x, y, Δh. The first three equations of the above group are ordinary differential equations of first order.

 [redacted] 25X1

Procedure for Numerical Integration of Differential Equations

44. The first force equation and the equations for the trajectory coordinates with the starting conditions $x = 0$, $y = 0$ for $t = 0$ were integrated in the Ballistics Section by Dr. WOLFF using the Bessel method. The reason was the following: in Bleicherode Dr. SCHLIER had worked with this system and the calculation personnel had learned it, because he had already used it in Peenemuende. Therefore, the use of this system was continued in Ostashkov and the old personnel were not forced to learn a new method.

45. In the working group of Prof. KLOSE integration was done using the method of Adams-Stoermer, probably because KLOSE was familiar with this method from experience in Germany.

46. The Soviet assistants who were assigned to us in Ostashkov (females with high school education) had no knowledge of the integration of differential equations, and were taught the Bessel method by us. They proved to be good calculators. Once, two computers came as help from Moscow. They used the Adams-Stoermer system and told us that this system was always used. [redacted] 25X1

47. The integration of the equations (1), (8), (9) was done in general with the interval $\Delta t = 2$ sec. When A-4 trajectories were not immediately calculated with full thrust, but with increase of the thrust up to full thrust, then $\Delta t = 0.5$ sec was selected in the first four to six seconds as an integration interval. But generally calculations were done with full thrust from the start.

48. Acceleration $\dot{v}$ was calculated to a hundredth of m/sec^2, velocity v to a tenth of m/sec, coordinates were calculated to the nearest meter. Later the mass ratio was often selected as an independent variable instead of the time t. This has several small advantages for the numerical calculations.[1]

Activity at "Gema", Berlin (July to October 1946) -- Mainly Concerned with "Wasserfall" Missile

49. [redacted] 25X1 Among the members of the group were a number of younger persons who had already worked during the war with Prof. KLOSE in Kummersdorf, especially a Dipl. Phys. STANGE and a Dipl. Chem. KLUGE. Several technical calculators and draftsmen served as assistants; there was also one stenotypist. The Soviet supervisors of the section were: Col. POKROWSKI and his deputy Lt. Col. SORKIN. [redacted] 25X1 [redacted] 25X1 POKROWSKI was a colonel-engineer, but had no specialized knowledge of mathematics or ballistics. SORKIN on the other hand was trained in ballistics, and had also occupied himself with rocket problems. [redacted] he worked at that time on questions of the mechanics and dynamics of bodies with variable mass. Except for these two, no other Soviets had contact with the section led by Prof. KLOSE. [redacted] 25X1

[redacted] 25X1

50. The task for the KLOSE group was the re-designing of the development of the AA-rocket "Wasserfall". The work was mainly in calculations. [redacted] the KLOSE group had been working on this task for several months. The reference data available to the KLOSE group was very scarce. Several design drawings of the whole rocket were available, but they contradicted each other in detail. KLOSE intended to attend to the ballistic problems himself, STANGE to the aerodynamic questions, and KLUGE to the thermodynamic problems. [redacted] no reports had yet been prepared by the KLOSE group. Especially unclear at that time was the problem of the guidance of the missile. Only several weeks after my arrival did SORKIN appear with an equation, which was given the designation: "differential equation of the ground calculating machine of the rocket 'Wasserfall'" - the applicable figure is shown below.

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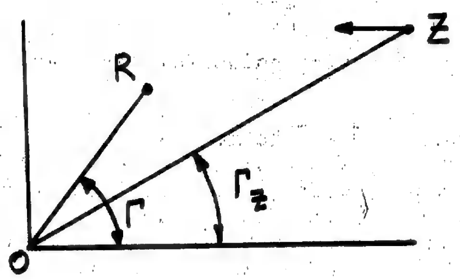

R = rocket

Z = target

O = launching point assumed identical with the ground station

Γ = angle of elevation, at which the rocket appears from the ground station, Γ_z corresponding angle for the target (aircraft)

Figure #4

51. The equation was a differential equation of second order for Γ which connects Γ with Γ_z. The exact form of the equation is no longer known to me. The following values appeared in the equation:

$\ddot{\Gamma}$, $\dot{\Gamma} - \dot{\Gamma}_z$ a function of $\Gamma - \Gamma_z$ and a function of t. The function of $\Gamma - \Gamma_z$ had the form, when $\Gamma - \Gamma_z = \delta$: $g(\delta) = \dfrac{a\delta}{1 + b\delta + c\delta^2}$ or similar, with constant values for a, b, c. This was valid when δ was not too great, [redacted] for $\delta \leq 30°$. For $\delta > 30°$, $g(\delta)$ was constant equal 3.9 o/sec (?) [redacted] the function of t was:

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$$f(t) = 12/(t-5)$$

52. The differential equation of the calculating unit should be valid from t = 6 sec on. Until this moment the rocket flew vertically upwards. At t = 6 sec, the change from the vertical flight began. The angle of elevation Γ of the rocket could be determined from the differential equation of the calculating unit. Heretofore, only the knowledge of the angle of elevation of the target was necessary. The differential equation was of a nature, that δ approached zero with increasing t, i.e., the deflecting arc changed over into the target seeking path by which ground station, rocket, and target lay in one line ($\Gamma = \Gamma_z$) As soon as the difference between Γ and Γ_z was less than 0.5°, it

should be calculated until the impact of the rocket on the the target-seeking procedure.

53. Thus, the flight path of the rocket consisted of three parts: (1) a vertical ascent in the first six seconds after the launching; (2) the arc of deflection determined by the calculating unit; and (3) the target seeking path. When the target approach lay in a vertical plane through the launching point, then the differential equation of the calculating unit is sufficient for determining the movement of the rocket. In any other movement of the target, further data is needed; for example, the side angle σ against a fixed direction in the horizontal plane, perhaps the x - direction. No information at all could be gained in this respect, neither at Gema in Berlin, nor later in the USSR. Other German working groups at Gema, which possibly had some knowledge about the calculating unit were not asked about this by KLOSE. SORKIN, who had given us the differential equation, could not say anything more. [redacted] not told about the make-up or the operation of the calculating unit. 25X1 Thus, one of the most important questions, namely under what command the rocket should fly in space, remained unsolved. Consequently, [redacted] worked in Berlin only with plane trajectories of the rocket "Wasserfall". 25X1

54. But several weeks passed before trajectory investigations for "Wasserfall" could start at all. [redacted] began only in the second half of September 1946 with a trajectory calculation. 25X1 Until then, even preliminary data on weights, thrust, weight rate of flow, gas exhaust velocity, etc., were missing.

55. During the first part of [redacted] work at Gema, weights and center of gravity location were calculated from old designs. 25X1 STANGE occupied himself with the theoretical determination of resistance and lift factors and the shift of center of pressure. KLUGE made theoretical examinations of the exhaust velocity of the combustion gases and of the thrust.

56. In the ballistic field, several general examinations were started at this time. Several target tracking procedures for antiaircraft rockets were examined:

a. The target seeking method. The rocket is guided from a ground station in such a way, that ground station O, rocket R and target Z are always on one line (see Figure No. 5).

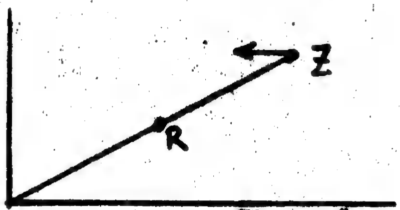

Figure #5

b. Generalization. The rocket is guided from the ground station in such a way that the angle between ground station, rocket, and target has a fixed value x (see Figure #6).

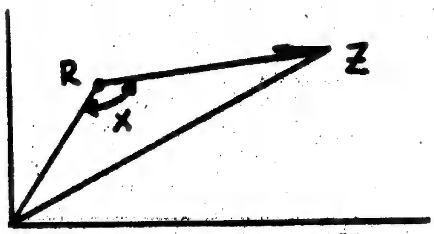

Figure #6

c. Dog curve. The rocket has a searcher head, and flew in such a way that its longitudinal axis always aims at the target. In mathematical calculations, the condition was also accepted that the trajectory tangent always points at the target (see Figure #7).

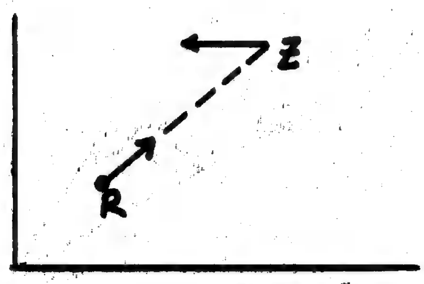

Figure #7

d. Dog curve with angle of lead. The rocket flies in such a way that its longitudinal axis (or the trajectory tangent) always forms a prescribed angle (angle of lead) with the connecting line from rocket to target (see Figure #8).

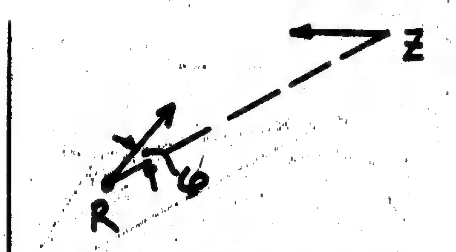

Figure #8

 [redacted]

57. For x = 180°, case b., above, becomes a.; for x = 0°, case d. becomes c. In these tasks the equations for the trajectory calculations were first formulated, and secondly the transverse accelerations appearing in each procedure were examined. 25X1

58. Prof. KLOSE made stability examinations for the first part of a vertical ascent after the start and used thus already provisional data for the "Wasserfall". [redacted] 25X1

Data for "Wasserfall"

59. [redacted] 25X1

O. The starting weight was about 3.8 tons, the thrust on the ground 8 to 8.3 tons, the burning period 46 seconds. The jet rudders were dropped off in the seventeenth or nineteenth second.

. The c_w values in the subsonic region (Ma < 0.8) for $\alpha = 0$ were approximately $c_w = 0.250$, the maximum of c_w (under Ma = 1.1 to 1.2) was 0.8 or more.

Motion Equations for a Plane AA Rocket Trajectory

, First the motion of the target is given. In the most simple case the aircraft flies in a vertical plane through the launching point of the rocket (x,y - plane), in constant altitude H with constant velocity V and in the direction of the negative x - axis (abscissa of the target equals X).

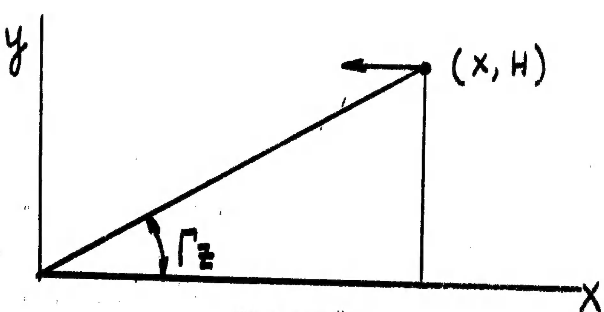

Figure #9

63. Then the cotangent of the angle of elevation is a linear function of the time:

$$\cot \Gamma_z = \frac{X}{H} = \frac{X_1}{H} - \frac{V}{H} t$$

$$\dot{\Gamma}_z = \frac{V}{H} \sin^2 \Gamma_z$$

$$\ddot{\Gamma}_z = 2 \frac{V}{H} \dot{\Gamma}_z \sin \Gamma_z \cos \Gamma_z$$

$$= 2 \dot{\Gamma}_z^2 \cot \Gamma_z$$

64. Therefore Γ_z and the time derivatives are known, and it is possible to calculate the angle of elevation Γ of the rocket and its derivatives from the differential equation of the calculating unit (or from $\Gamma = \Gamma_z$ if the rocket flies with the target-seeking method). The starting conditions are $\Gamma = 90°, \dot{\Gamma} = 0$ for t = 6 sec. $\Gamma, \dot{\Gamma}, \ddot{\Gamma}$ can therefore be regarded as given. For the treatment of the motion equations (1), etc. it is evident to use polar coordinates :

(12) $x = r \cos \Gamma$

(13) $y = r \sin \Gamma$

Differentiation results:

(14) $\dot{x} = \dot{r} \cos \Gamma - r \dot{\Gamma} \sin \Gamma$

(15) $\dot{y} = \dot{r} \sin \Gamma + r \dot{\Gamma} \cos \Gamma$

It follows:

(16) $v^2 = \dot{x}^2 + \dot{y}^2 = \dot{r}^2 + r^2 \dot{\Gamma}^2$

(17) $v\dot{v} = \dot{r}\ddot{r} + r\dot{\Gamma}(\dot{r}\dot{\Gamma} + r\ddot{\Gamma})$

If the first force equation is multiplied with v, then there follows:

(18) $m v \dot{v} = m \{\dot{r}\ddot{r} + r\dot{\Gamma}(\dot{r}\dot{\Gamma} + r\ddot{\Gamma})\}$

$= Sv - Wv - mgv \sin \gamma$

v. can hereby be expressed by (16) by the polar coordinates and their derivatives. The same hold for $v \sin \gamma$ based on the equations (9) and (15).

(19) $v \sin \gamma = \dot{r} \sin \Gamma + r \dot{\Gamma} \cos \Gamma$

66. The right side of (18) also depends upon the altitude (because of S, W, g). It is possible to take y for the altitude in case of AA rockets with little distances from the launching point, which can be reduced by (13) to r and Γ . There with (18) represents a differential equation of second order for r, and r is the only unknown. ($\dot{r}$ is at the beginning of the calculation different from zero, since the starting values are the values for t = 6 sec in the vertical ascent.)

67. By calculating r through integration of (18), v is also given through (16) and y through (13). The abscissa x is found through (12) and the trajectory angle of inclination γ can finally be determined through (19).

68. Appropriate formulas for calculation of the trajectory angle of inclination γ can also be found in another way:

$\tan \Gamma = y/x$ or $x \sin \Gamma - y \cos \Gamma = 0$

Differentiation results in

$\dot{x} \sin \Gamma + x \dot{\Gamma} \cos \Gamma - \dot{y} \cos \Gamma + y \dot{\Gamma} \sin \Gamma = 0$

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By consideration of (8), (9), (12), and (13) follows:

$$v(\cos\gamma\sin\Gamma - \sin\gamma\cos\Gamma) + \dot{\Gamma}r(\cos^2\Gamma + \sin^2\Gamma) = 0$$

or (20) $v\sin(\gamma - \Gamma) = r\dot{\Gamma}$

Correspondingly follows from $x\cos\Gamma + y\sin\Gamma = r$ through differentiation

(21) $v\cos(\gamma - \Gamma) = \dot{r}$

From (20) and (21) follows by division

(22) $\tan(\gamma - \Gamma) = \frac{r\dot{\Gamma}}{\dot{r}}$

69. Each of the formulas (19), (20), (21), (22) can be used for the calculation of γ. In the second force equation appears the derivative $\dot{\gamma}$, which is found through differentiation of one of the formulas (19) to (22); for example from the formula (20):

$$\dot{v}\sin(\gamma - \Gamma) + v(\dot{\gamma} - \dot{\Gamma})\cos(\gamma - \Gamma) = \dot{r}\dot{\Gamma} + r\ddot{\Gamma}$$

or on account of (20) and (21):

$$\frac{\dot{v}r\dot{\Gamma}}{v} + \dot{r}\dot{\gamma} - \dot{r}\dot{\Gamma} = \dot{r}\dot{\Gamma} + r\ddot{\Gamma}$$

that is

(23) $v\dot{\gamma} = \left(2v - \frac{r\dot{v}}{\dot{r}}\right)\dot{\Gamma} + \frac{rv}{\dot{r}}\ddot{\Gamma}$

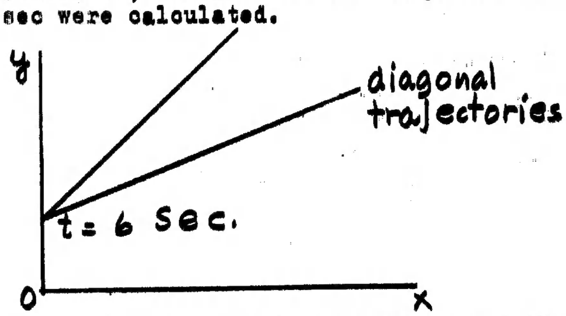
This is an exact formula for the transverse acceleration.

70. Angle of incidence and rudder angle are to be calculated by the equations (4) to (7).

"Wasserfall" Calculations at Gema (until 22 October 1946)

71. When approximate preliminary data for weight, thrust and resistance coefficients were known, several vertical ascents were calculated, based on different values for starting weight and starting thrust.

72. Furthermore, linear inclined trajectories with starting values for t = 6 sec were calculated.



73. Since lift coefficients were known and the function of the resistance coefficients with the angle of attack and the rudder angle were determined, the influence of the c_w increase on the burning out-off speed and the burning out-off location was examined. The dependence of the c_w value on α and η was formulated:

$$c_W = c_{W_0} + c_W^{(1)}\alpha^2 + c_W^{(2)}\eta^2 + c_W^{(3)}\alpha\eta.$$

This equation had been used previously in Gema.

25X1

74. The increase of the C_w -value by considering the dependency on α and η was very substantial, even if the beginning of the transverse trajectory was neglected. The beginning of the transverse trajectory demands by comparison with normal trajectories with slow deflection from the vertical ascent; very great angles of incidence.

[redacted] 25X1

75. The result was: in transverse trajectories the air resistance becomes so much greater, that the gain on cut-off velocity by the more favorable weight component $mg \sin \gamma$, in comparison to the vertical ascent, is lost again. This result was later confirmed by more accurate trajectory calculations [redacted] in the USSR. 25X1

76. In steep and also in level trajectories the cut-off velocity was about 800 m/sec or slightly more. The consideration of the strong dependency of the c_w value on α and η produced the result that the equations (4) and (6) had to be solved simultaneously with the equation (18).

77. The differential equation of the calculating unit was examined, to determine when the deflection arc is completed and the target approach begins. The result is that the point of time can be rather late in some approaches ($\approx$ 38 sec.). The target approach path would in such a case only begin in rather high altitude ($\approx$ 8 to 9 kilometers) and a target which flies at low altitude could not be attacked with the rocket "Wasserfall" for such an approach.

78. In the second half of September 1946 a trajectory calculation was done by utilizing the differential equation of [redacted] 25X1

79. In the first half of October 1946 SORKIN asked Prof. KLOSE to make reports on the current research. This was done [redacted] saw these reports 25X1 again in the spring of 1947 in Ostashkov. They were compiled in three volumes with the following titles:

a. Institute Berlin: Ballistics of the Rocket Wasserfall.

b. Institute Berlin: Aerodynamics of the Rocket Wasserfall.

c. Institute Berlin: Thermodynamics of the Rocket Wasserfall.

The first volume contained Prof. KLOSE's and [redacted] examinations, the 25X1 second contained STANGE's examinations and also design data, the third was the work of KLUGE.

80. On the 22nd of October Prof. KLOSE [redacted] sent to the USSR. The 25X1 remainder of the co-workers of KLOSE's group remained in Germany and were, shortly after [redacted] departure, given notice at Gema. STANGE, and 25X1 perhaps a few others, were then still working for a short time closing the office. Neither STANGE nor KLUGE went to the USSR.

[redacted]

25X1